RBPO
REVISTA BRASILEIRA DE
PLANEJAMENTO
E ORÇAMENTO

Public debt dynamics in emerging economies: an accounting model based on the Brazilian case

Nelson Barbosa nelson.barbosa@fgv.br
Professor da Escola de Políticas Públicas e Governo da Fundação Getúlio Vargas.

Governments face a fiscal constraint in the form of a non-explosive debt to GDP ratio. The restriction is usually more binding in emerging economies because they do not issue an international reserve currency and often have a sizeable part of their public debt indexed to foreign money. Since the early 2000s, many countries responded to this asymmetry by accumulating a large stock of international reserves, giving their central bank more room to attenuate exchange-rate volatility through swap operations.

This note adapts the usual accounting model of public-debt dynamics (Escolano 2010) to a government that issues debt and accumulates financial assets in two currencies, domestic and international (Barbosa-Filho 2014 and 2021). The model includes the dynamics of gross and net debt ratios, including the capital gains or losses caused by exchange-rate variations. The accounting identities can be closed by more than one theoretical model, based on behavioral assumptions (optimization or policy rules) and stylized facts (empirical regularities). The analysis is in three sections.

1 – The intertemporal budget constraint

Assume the national Treasury and the Central Bank form a single unit because the Treasury is the Central Bank's sole shareholder. In this case, a country can never default on its domestic currency. If debt holders do not want to roll over the Treasury's debt, it is theoretically possible to pay all bonds that come due by "just" printing money. Basic accounting confirms this double-entry fact, but it does not shed much light on debt-monetization consequences.

If society (people and firms) demand more money, public debt can be monetized without problem, that is, without creating a run against the domestic currency. However, suppose the private sector does not want more money. The looming debt monetization may start a speculative attack against the domestic currency, usually a flight to an international currency. The resulting increase in the exchange rate, defined as the foreign currency's domestic price, raises inflation before monetization even starts. The money stock falls in real terms, while inflation accelerates, in a hyperinflation driven by self-fulfilling expectations.

To analyze the expectational constraint on debt monetization, consider a closed economy where the change in government debt (D_t) and money (M_t) depends on the government's primary balance ($B_{p,t}$) and implicit nominal interest rate of government debt at the end of the previous period (i_{t-1}):

$$D_t + \Delta M_t = -B_{p,t} + i_{t-1} D_{t-1} \ . \quad (1.1)$$

Following the conventional approach of monetary economics (Godley and Lavoie 2011, Walsh 2017), note that I assumed that the nominal interest rate i_{t-1} is fixed at the end of the period t-1, and paid at the end of period t. The length of the period can be a year, month, week, or even a day, depending on the data's frequency.

From (1.1), after some algebraic operations,[1] the dynamics of the ratio of public debt to GDP (d_t) is:

$$d_t = -b_{p,t} + \left(\frac{1+r_t}{1+g_{Y,t}}\right) d_{t-1} - \left(\frac{g_{M,t}}{1+g_{PY,t}}\right) m_{t-1} \quad (1.2)$$

where $B_{p,t}$ is the primary balance in terms of GDP, r_t the effective (ex-post) real implicit interest rate on government debt, according to the GDP deflator, $g_{Y,t}$ the growth rate of real GDP, $g_{M,t}$ the growth rate of money, $g_{PY,t}$ the growth rate of nominal GDP, and m_t the ratio of money to GDP.

Assuming the cost of printing money is null, the last term on the right-hand side of (1.2) is the government's seigniorage, the government revenue from the monopoly of issuing the country's official means of payment (Buiter 2007). In practice, the cost of printing money is low and registered as an expenditure of the Central Bank in the government's primary balance. Because of this, I will proceed under the simplifying assumption that seigniorage is the change in money supply (monetary base).

To simplify exposition, define seigniorage in terms of GDP as s_t and rewrite (1.2) in a forward-looking manner:

$$d_t = \left(\frac{1+g^e_{Y,t}}{1+r^e_{t+1}}\right)\left(d^e_{t+1} + b^e_{p,t+1} + s^e_{t+1}\right). \quad (1.3)$$

The intuitive meaning of (1.3) is that debt today equals the present value of the expected sum of debt, primary surplus, and seigniorage tomorrow. The discount rate is "r minus g"; that is, the cost of public finance is the difference between the expected real interest rate paid by government bonds and the economy's growth rate.

Now assume, again for simplicity, that there exists an average real interest rate (r^e), real GDP growth rate (g^e_Y), primary balance (b^e_p), and seigniorage (s^e) that summarizes the agents' expectations for the following N periods. In this case, we can iterate (1.3) forward to obtain:

1 The annex presents all derivations.

$$d_t = \sum_{i=1}^{N} \left(\frac{1+g_Y^e}{1+r^e}\right)^j (b^e + s^e) + \left(\frac{1+g_Y^e}{1+r^e}\right)^N d_{t+N}^e. \tag{1.4}$$

If the expected real rate of interest is larger than the expected growth rate of GDP ($r^e > g_Y^e$), what economists call the "dynamic efficiency condition" (Barro 2020), the last term on the righthand side of (1.4) vanishes as N tends to infinity. In this case, we have:

$$d_t = (b^e + s^e)\left(\frac{1+g_Y^e}{r^e - g_Y^e}\right). \tag{1.5}$$

In other words, the debt ratio is the present value of future primary surpluses and seigniorage, with the discount rate given by "r minus g".

According to (1.5), if everything else remains equal, a higher debt ratio requires a higher expected primary surplus. However, suppose there is an upper limit to the primary surplus the government can obtain. In that case, the economy may goes into "fiscal dominance" (Blanchard 2004), the situation in which the expected real interest rate or seigniorage becomes whatever is necessary to satisfy (1.5) for a given GDP growth rate.

Specifically, assume that an exogenous shock, say, a global pandemic like Covid-19, pushes the debt-GDP ratio abruptly up. If the expected seigniorage, GDP growth, and real interest rate do not change, the adjustment must come from the expected primary surplus. The government will have to "save" more in the future to "pay for" the increase in its debt today. However, if the required increase in the primary surplus is more than the "market" expects "society" is willing to accept, the adjustment of expectations will come from somewhere else. From (1.5) the candidates are:

(i) a faster growth rate,
(ii) a lower real interest rate,
(iii) more seigniorage,
(iv) a unilateral reduction in the debt ratio through full or partial default, or
(v) a discontinuous jump in the price level today.

Fiscal dominance happens in cases (ii) and (iii), since they mean that the interest rate and inflation become determined by budgetary issues, not by monetary concerns about inflation targets. As we mentioned, the process's logic is that a looming public debt crisis triggers a speculative attack against the domestic currency. Exchange-rate depreciation then pushes inflation up and the real interest rate down, which in turn "solve" the government intertemporal fiscal constraint, but not in a favorable way to public-debt holders. Case (v) is the extreme case described by the fiscal theory of the price level, the idea that debt sustainability determines prices, an adaptation of the quantity theory of money to the 21st century, with the stock of debt in place of the stock of money.

Fiscal dominance is one possible case, not the only possible case after a sudden increase in public debt. The intertemporal constraint given by (1.5) can also be satisfied by a non-inflationary fall in the real interest rate (the secular-stagnation hypothesis), a non-inflationary increase in the country's money holdings (the liquidity--trap idea), or faster income growth (the usual Keynesian prescription for a country to grow his way out of debt).

Under normal conditions, monetary policy dominates fiscal policy. The intertemporal budget constraint gives us the average expected primary surplus compatible with the economy's growth rate and the real interest rate necessary to meet the government's inflation target with little or no risk of speculative attack against the domestic currency. The GDP growth rate and the inflation target fix seigniorage, and the primary surplus adapts to the real interest rate. Under fiscal dominance, the roles are reversed.

For practical matters, the intertemporal identity allows us to analyze the risk of fiscal dominance. For example, consider the case of Brazil at the end of 2020. Assuming the long-run inflation target and GDP growth rate are 3% and 2%, respectively, a money-GDP ratio of 5% means that expected seigniorage is:

$$s^e = \left(\frac{0.03+0.02+0.03\times0.02}{1+0.03+0.02+0.03\times0.02}\right)0.05 = 0.24\% \text{ of GDP}$$

Considering the General Government (Federal, State, and Municipal) and the Central Bank, Brazil's net public debt was 62.2% of GDP. Assuming a long-run annual real interest rate of 4%, the Brazilian intertemporal expectational identity was:

$$0.622 = (b^e + 0.24)\left(\frac{1+0.02}{0.04-0.02}\right).$$

In other words, from 2021 until the "end of time", the Brazilian government would have to register an average primary balance of 1% of GDP to exclude the possibility of a speculative attack against its currency at any point in the future. What if the expected long-run balance is lower than 1% of GDP? People expect a fiscal crisis, sometime between 2021 and "forever", to close the expectational identity.

The primary balance necessary to close the expectational identity without a fiscal crisis depends on the state of long-term expectations: the average expected GDP growth and real interest rate. Table 1 shows how small changes in this kind of expectations alter the expected fiscal effort implicit in (1.5). For example, if the real interest rate is 3% instead of 4%, the Brazilian government's required balance falls to 0.38% of GDP from 2021 through infinity. If the real interest rate remains at 4% but expected growth increases to 3%, the required balance falls to 0.33% of GDP. The lower the "r minus g", the lower the required balance to avoid a speculative attack from tomorrow until infinity.

Table 1: example of required annual primary balance, in terms of GDP, according to the intertemporal constraint with infinite time.

Real GDP growth	Real interest rate					
	2.5%	**3.0%**	**3.5%**	**4.0%**	**4.5%**	**5.0%**
2.0%	0.07%	**0.38%**	0.69%	**1.00%**	1.31%	1.62%
2.5%	-0.26%	0.04%	0.35%	0.66%	0.97%	1.28%
3.0%	-0.59%	-0.29%	0.02%	**0.33%**	0.63%	0.94%
3.5%	-0.92%	-0.62%	-0.31%	0.00%	0.30%	0.61%
4.0%	-1.24%	-0.94%	-0.64%	-0.33%	-0.03%	0.28%

Source: author's calculation, based on an initial debt ratio of 63.2%, a money-GDP ratio of 5%, and constant annual inflation at 3%

Now, what is an infinite interval of time? The long run is a valuable concept for academic theorems. In the real world, one needs to gauge a speculative attack's risk during a definite time interval. To see how we can do this, go back to the intertemporal identity, but use (1.4) instead of (1.5). Based on the Brazilian case outlined earlier, suppose the average market expectation is that net public debt must fall to 60% of GDP in ten years for the country not to suffer a speculative attack.

Assuming average inflation to be 3% and the money-GDP ratio 5% as before, table 2 gives us the required primary balance for alternative 10-year scenarios of growth and real interest rates. Once more, the required primary balance varies substantially with expectations and, since we used a finite time horizon in table 2, we can now allow "r minu g" to be negative.

Table 2: example of the required annual primary balance, in terms of GDP, according to the intertemporal constraint with finite time.

Real GDP growth	Real interest rate					
	1.50%	**2.00%**	**2.50%**	**3.00%**	**3.50%**	**4.00%**
1.5%	0.32%	0.41%	0.71%	1.02%	1.32%	1.63%
2.0%	-0.22%	0.32%	0.38%	0.68%	0.99%	**1.29%**
2.5%	-0.55%	-0.25%	0.32%	0.36%	0.66%	0.96%
3.0%	-0.87%	**-0.57%**	-0.27%	**0.32%**	0.33%	0.63%
3.5%	-1.18%	-0.88%	-0.59%	-0.29%	0.32%	0.31%
4.0%	-1.50%	-1.20%	-0.90%	-0.61%	-0.31%	0.32%

Source: author's calculation, based on an initial debt ratio of 63.2%, a final debt ratio of 60% after 10 years, a money-GDP ratio of 5%, and constant annual inflation at 3%

Translating table 2 in words, for the Brazilian net public debt to reach 60% in 2030, the average primary balance will have to be 1.29% of GDP in the scenario with a 2% GDP growth and 4% real interest rate. If GDP growth jumps to 3% and the interest rate falls to 3%, the required primary balance falls to 0.32% of GDP.[2] And, if expected GDP growth remains at 3% but the real interest rate is 2%, the required balance drops to a deficit of 0.57% of GDP. The "r minus g" is negative in the latter case, meaning time itself erodes part of the debt burden, the modern "debt jubilee", currently restricted to a few economies.

A long time ago, Domar (1943) pointed out that small changes in "r minus g" heavily influence the primary balance necessary to meet a given debt target. This influence is often downplayed by mainstream macro models, where "r" and "g" come from technology and preferences and, therefore, are not affected by short-run changes in fiscal and monetary policy. The evidence since the Global Financial Crisis of 2008 showed that the long run is not that stable as mainstream models used to assume. Short-run shocks may have permanent effects on the economy (hysteresis), meaning that they cease to be short-run. In terms of the intertemporal identity analyzed above, the long-run value of "r minus g" becomes a function of the monetary and fiscal policy, which leads to multiple equilibria.

2 – Debt dynamics with fixed-income assets

The previous section analyzed a government that issues debt to finance a primary deficit and pay interest on its debt. However, in the real world, governments also hold financial fixed-income assets. This extension is interesting for some emerging economies, where the Treasury has quotas in "investment vehicles" and lends funds through public (national banks) and private financial agents.[3] And even for advanced economies, it is worthy to model the accumulation of fixed-income assets explicitly when the Treasury's account balance at the Central Bank is macro-economically large.

From an accounting perspective, introducing government fixed-income assets in the model creates two public--debt concepts: net and gross, with possibly different implications for fiscal adjustment.

More formally, when the government can accumulate fixed-income assets, the budget identity changes to:

$$\Delta D_t + \Delta M_t = -B_{p,t} + i_{D,t-1}D_{t-1} - i_{A,t-1}A_{t-1} + \Delta A_t \tag{2.1}$$

where i_A is the nominal interest rate on the government's fixed-income assets .

The intuitive meaning of (2.1) continues to be that the government issues debt and money to cover a primary deficit and pay interest. The novelty is that now the government can also use debt or money to buy fixed-income assets, which means that its interest revenues pay part of its interest expenditures.

2 Still on table 2, note that whenever "r minus g" is zero, the primary balance is just the expected debt reduction divided by the number of years during which people expect this to happen.

3 Following the IMF (2014) methodology, acquisition of real and variable-income assets is recorded as an expenditure in the government's primary balance.

If we define net debt (D_N) as

$$D_{N,t} = D_t - A_t, \tag{2.2}$$

we can rewrite (2.1) as:

$$\Delta D_{N,t} + \Delta M_t = -B_{p,t} + i_{N,t-1} D_{N,t-1}$$

where $i_{N,t-1}$ is the implicit interest rate on net debt, that is:

$$i_{N,t-1} = \frac{i_{D,t-1} D_{t-1} - i_{A,t-1} A_{t-1}}{D_{t-1} - A_{t-1}} = i_{D,t-1} + \left(i_{D,t-1} - i_{A,t-1}\right) \frac{A_{t-1}}{D_{t-1} - A_{t-1}}. \tag{2.3}$$

In words, assuming the government is a net debtor ($D > A$), the accumulation of fixed-income assets raises the cost of net debt (i_N) above the cost of gross debt (i_D) when the government pays more on its liabilities than it earns on its assets ($i_D > i_A$). The latter is the usual case since governments tend to engage in financial intermediation to lend at subsidized interest rates to activities or regions of strategic importance to its policies.

Equation (2.3) helps calculate the carry cost of government assets in developing countries where, in addition to its usual fiscal activities, the Treasury also acts as a financial intermediary to compensate for market failures.

From (2.2) and (2.3), the dynamics of the ratio of net debt to GDP is:

$$d_{N,t} = -b_{p,t} + \left(\frac{1 + r_{N,t}}{1 + g_{Y,t}}\right) d_{N,t-1} - s_t \tag{2.4}$$

where we used the fact that $\left(1 + i_{N,t-1}\right) = (1 + r_{N,t})(1 + g_{P,t})$.

In contrast to the previous section, changes in the government debt-asset portfolio can alter the cost of net debt in (2.4) even if the underlying interest rates on government debt and assets remain the same.

Moving to gross debt, from (2.1) we have:

$$d_t = -b_{p,t} + \left(\frac{1 + r_{D,t}}{1 + g_{Y,t}}\right) d_{t-1} + \left(\frac{g_{A,t} - i_{A,t-1}}{1 + g_{PY,t}}\right) a_{t-1} - s_t. \tag{2.5}$$

where $g_{A,t}$ is the nominal growth rate of the government's fixed-income assets and $r_{D,t}$ the implicit interest rate on gross debt.

The economic meaning of (2.5) is that, in addition to the primary balance, interest payments, economic growth, and seigniorage, gross debt also depends on the accumulation of financial assets. For example, if the growth rate of government assets is higher than the return on these assets ($g_A > i_A$), gross debt goes up even if everything else in the righthand side of (2.5) adds up to zero.

Finally, from (2.4), the primary surplus that stabilizes net debt is:

$$b_{p,t}^{*N} = \left(\frac{r_{N,t}-g_{Y,t}}{1+g_{Y,t}}\right) d_{N,t-1} - s_t, \tag{2.6}$$

whereas from (2.5) the primary surplus that stabilizes gross debt is:

$$b_{p,t}^{*D} = \left(\frac{r_{D,t}-g_{Y,t}}{1+g_{Y,t}}\right) d_{t-1} + \left(\frac{g_{A,t}-i_{A,t-1}}{1+g_{PY,t}}\right) a_{t-1} - s_t. \tag{2.7}$$

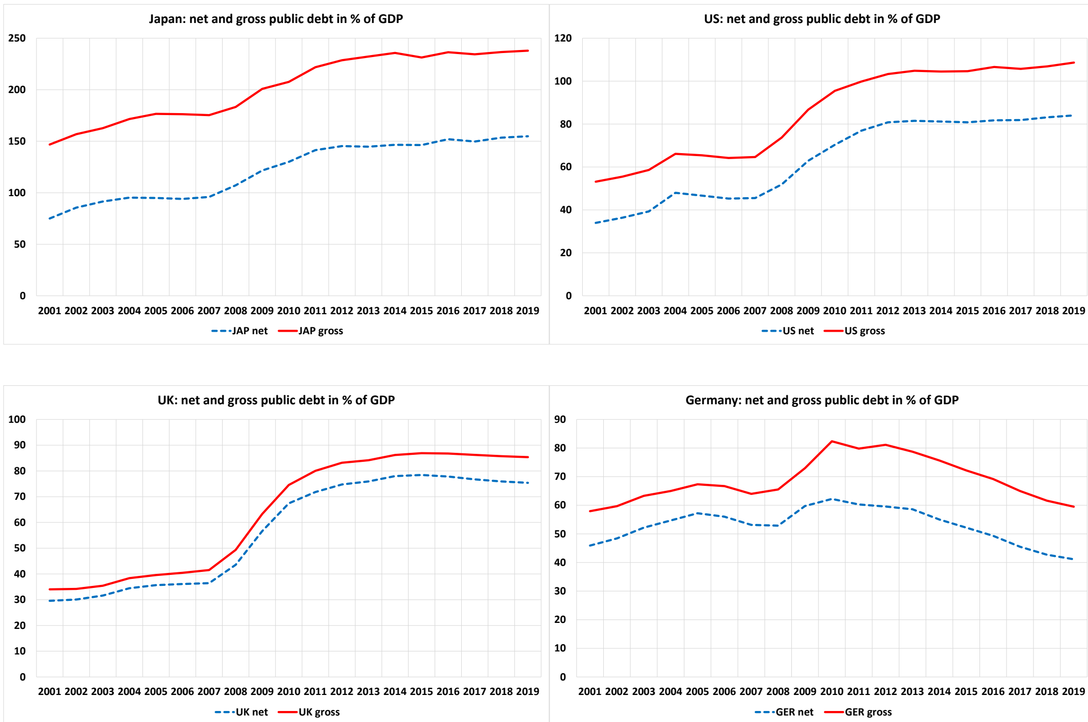
The difference between the two values is, therefore:

$$b_{p,t}^{*N} - b_{p,t}^{*D} = \left(\frac{g_{PY,t}-g_{A,t}}{1+g_{PY,t}}\right) a_{t-1}. \tag{2.8}$$

In words, the primary balance that stabilizes the net debt-GDP ratio is smaller than the primary balance that stabilizes gross debt-GDP ratio when the government is accumulating fixed-income assets faster than the economy is growing ($g_{PY,t} < g_{A,t}$). By analogy, (2.8) tells us that the difference between gross and net debt grows when the government is buying fixed-income assets faster than GDP growth ($g_{A,t} > g_{PY,t}$) and vice versa.

Now, how relevant is the accumulation of fixed-income assets by the government in the real world? Starting with advanced economies, figure 1a through 1d shows the IMF data for the US, Japan, UK, and Germany in 2001-19. The numbers cover only the General Government and, therefore, most of the government's assets consist of the Treasury's deposits at the Central Bank. The net and gross debt series move together in the four countries, meaning that the government's fixed-income assets are stable in terms of GDP. Because of the latter, one can use just one of the series, usually the gross one, to analyze fiscal scenarios.

Figure 1: net and gross debt ratios

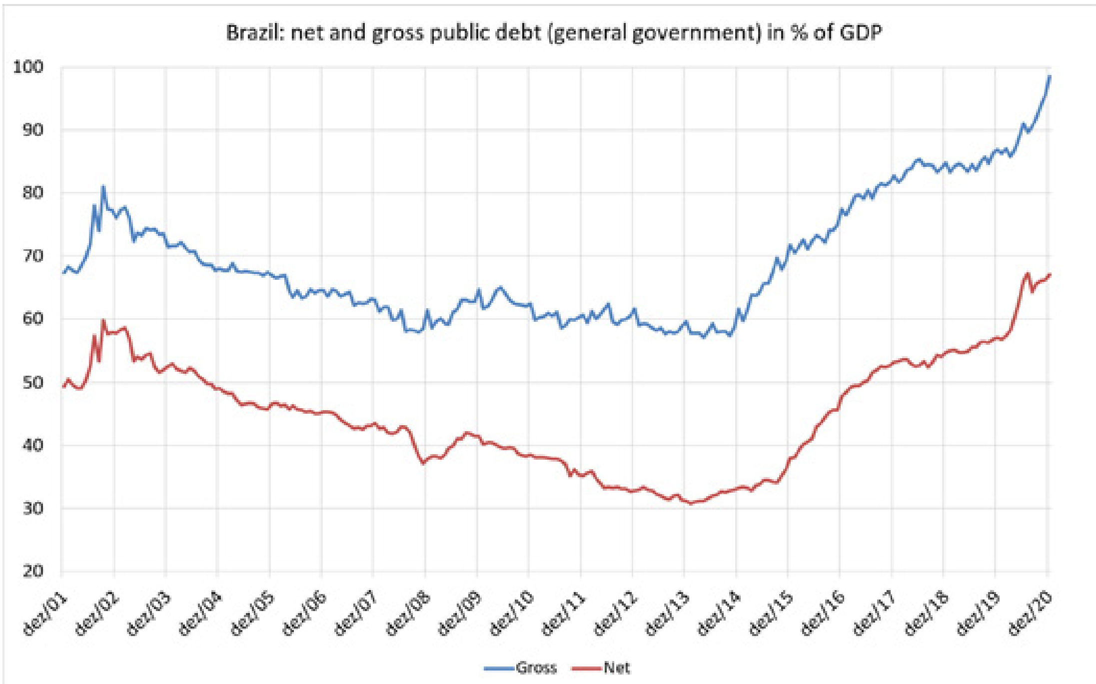


Source: IMF, WEO Oct/2020, numbers for 2019 are the IMF's projections

Despite the joint fluctuation of the net and gross debt series in figure 1, it is worthy mentioning that the size of the government's fixed-income assets varies substantially across the four countries depicted in it. For example, considering 2019, the IMF estimates that the Japanese government had 83% of Japan's GDP in fixed income assets, making its net debt substantially lower than gross debt. In contrast, in the UK, the same variable was "just" 10% of GDP, with the US and Germany in an intermediary position with 25% and 18% of GDP, respectively.

Moving to developing economies, let us again consider the case of Brazil. As shown in figure 2, the net and gross debt series of the General Government did not necessarily move together in the recent past. Specifically, the net and gross debt fell together from the early 2000s through the eve of the Global Financial Crisis of 2008. Then, as the Brazilian authorities increased the government lending through the national development bank, gross debt remained stable, while net debt continued to fall. The divergence persisted until the mid-2010s, when the two series resumed moving together.

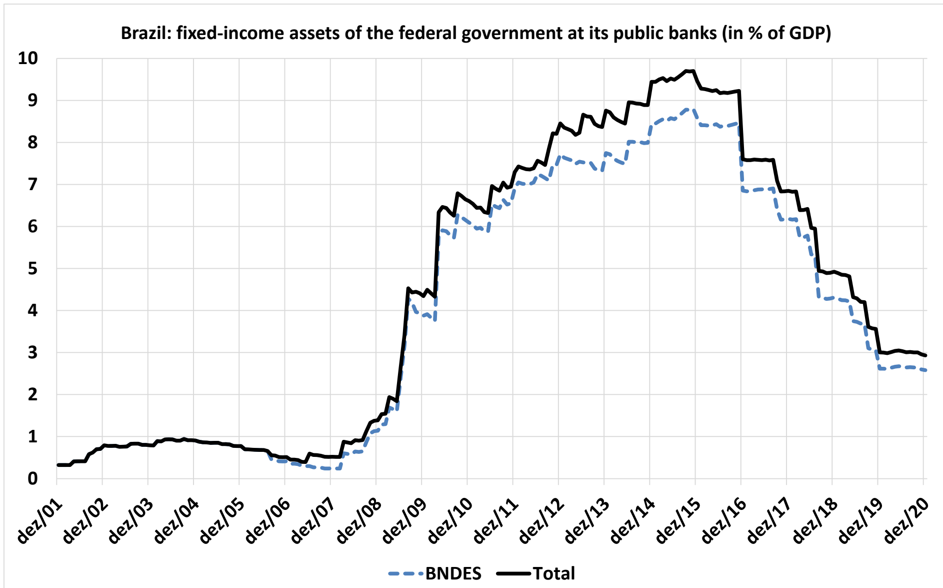
Figure 2: gross and net debt of the general government in Brazil



Source: Brazilian central bank, using the IMF's definition of gross debt

The difference between the two lines in figure 2 represent the Brazilian government's (Federal, State and Municipal), fixed-income assets. Like in developing countries, most of it consists of the National Treasury's deposits at the Brazilian Central Bank, but there was also a substantial value invested by the federal government in its public banks during the period under analysis. In fact, as shown in figure 3, the federal loans to Brazil's public banks grew to almost 10% of the country's GDP in the mid-2010s, which was the Brazilian way to do quantitative easing after the Global Financial Crisis of 2008. Most the resources went to Brazil's national development bank, BNDES, which in turn started paying them back to the National Treasury in 2016. A fiscal analysis focused solely on the primary balance tends to ignore the details of this kind of financial operation because its effect appears only indirectly as a change in the implicit interest rate on the government net debt.

Figure 3: Brazilian federal fixed-income assets in the country's public banks



Source: Brazilian Central Bank

Going back to figure 2, during the whole period, abrupt and substantial changes in the exchange rate, at the end of 2008 and the beginning of 2020, made a short-run dent in the Brazilian net debt series, due to the capital gains from the government's foreign exchange assets. The latter shows the importance modeling foreign assets and liabilities explicitly in the debt trajectory of developing economies, which is our next topic.

3 – Exchange-rate complications

Debt dynamics become more complicated when the government can issue liabilities and acquire assets in different currencies. To see why this happens, consider the simplest case with just two currencies, home (h) and foreign (f). The debt identity now becomes:

$$\Delta D_{h,t} + E_t \Delta D_{f,t} + \Delta M_t = -B_{p,t} + i_{Dh,t-1} D_{h,t-1} + E_t i_{Df,t-1} D_{f,t-1} - i_{Ah,t-1} A_{h,t-1} - E_t i_{Af,t-1} A_{f,t-1} + \Delta A_{h,t} + E_t \Delta A_{f,t}, \quad (3.1)$$

where E_t is the nominal exchange rate, defined as the domestic price of foreign currency, D_j represents debt in currency "j", A_j the government's fixed-income assets in currency "j", and i_j is the interest rate on debt or asset in currency "j".

To separate interest effects from exchange-rate gains or losses, it is helpful to define the net interest paid by the government as the net payments that would have occurred at a constant exchange rate. Formally, define:

$$J_t = i_{Dh,t-1} D_{h,t-1} - i_{Ah,t-1} A_{h,t-1} + +E_{t-1}(i_{Df,t-1} D_{f,t-1} - i_{Af,t-1} A_{f,t-1}), \tag{3.2}$$

so that we can rewrite debt dynamics as

$$\Delta D_{h,t} + E_t \Delta D_{f,t} + \Delta M_t = -B_{p,t} + J_t + Z_{J,t} + \Delta A_{h,t} + E_t \Delta A_{f,t}, \tag{3.3}$$

where

$$Z_{J,t} = g_{E,t} E_{t-1}(i_{Df,t-1} D_{f,t-1} - i_{Af,t-1} A_{f,t-1}) \tag{3.4}$$

is the effect of a depreciation of the home currency ($g_{E,t}$) on the government's net interest payments (the "flow effect" of depreciation).

The net debt now becomes:

$$D_{N,t} = D_{h,t} - A_{h,t} + E_t(D_{f,t} - A_{f,t}), \tag{3.5}$$

while the implicit interest rate on net debt is:

$$i_{N,t-1} = \frac{J_t}{D_{N,t-1}}. \tag{3.6}$$

Using (3.3), (3.5), and (3.6), the evolution of the net debt ratio is:

$$D_{N,t} = -B_{p,t} + J_t + Z_{J,t} + D_{N,t-1} + Z_{N,t} - \Delta M_t, \tag{3.7}$$

where

$$Z_{N,t} = g_{E,t} E_{t-1}(D_{f,t-1} - A_{f,t-1}) \tag{3.8}$$

is the impact of depreciation on net foreign debt (the "stock effect" of depreciation).

From the previous definitions and after some algebraic operations, the evolution of the net debt ratio is:

$$d_{N,t} = -b_{p,t} + \left(\frac{1+r_{N,t-1}}{1+g_{Y,t}}\right) d_{N,t-1} - s_t + z_{J,t} + z_{N,t}. \tag{3.9}$$

where $z_{J,t}$ and $z_{N,t}$ represent (3.4) and (3.5) divided by GDP, respectively. In countries with a substantial amount of foreign debt and assets, these two variables tend to dominate debt dynamics in the short run, as we will illustrate later for Brazil.

Moving to the dynamics of gross debt, the previous definitions and another set of algebraic operations allow us to define:

$$d_t = -b_{p,t} + \left(\frac{1+r_{D,t-1}}{1+g_{Y,t}}\right) d_{t-1} + \left(\frac{g_{Ah,t}-i_{Ah,t-1}}{1+g_{PY,t}}\right) a_{h,t-1} + \left(\frac{g_{Af,t}-i_{Af,t-1}}{1+g_{PY,t}}\right) a_{f,t-1} - s_t + z_{J,t} + z_{G,t} \tag{3.10}$$

where now g_{Ah} and g_{Af} represent the nominal growth rates of the government's fixed-income assets in home and foreign currency, respectively, and

$$z_{G,t} = \frac{g_{E,t}E_{t-1}(g_{Af,t}a_{f,t-1}+d_{f,t-1})}{1+g_{PY,t}}$$

is the impact of depreciation (g_E) on gross debt normalized by GDP.

Compared with the simpler model of the previous section, (3.10) adds two new factors to the evolution of gross debt: the pace of accumulation of foreign assets and the impact of exchange-rate variations on the acquisition of foreign investments and the previous stock of foreign debt. The two issues are essential for countries that aim to maintain a high stock of international reserves, keeping it proportional to their GDP of gross debt.

For example, focus on the fourth term on the right-hand side of (3.10). If the country aims to keep its international reserves stable in terms of GDP, we have:

$$(1 + g_{Af,t})(1 + g_{E,t}) = (1 + g_{P,t})(1 + g_{Y,t})$$

or

$$1 + g_{Af,t} = \frac{1+g_{Y,t}}{1+g_{Q,t}}(1 + g_{Pf,t}) \tag{3.11}$$

where $g_{Q,t}$ is the growth rate of the real exchange rate (the nominal rate multiplied by the ratio of foreign-home price levels) and $g_{Pf,t}$ is foreign inflation.

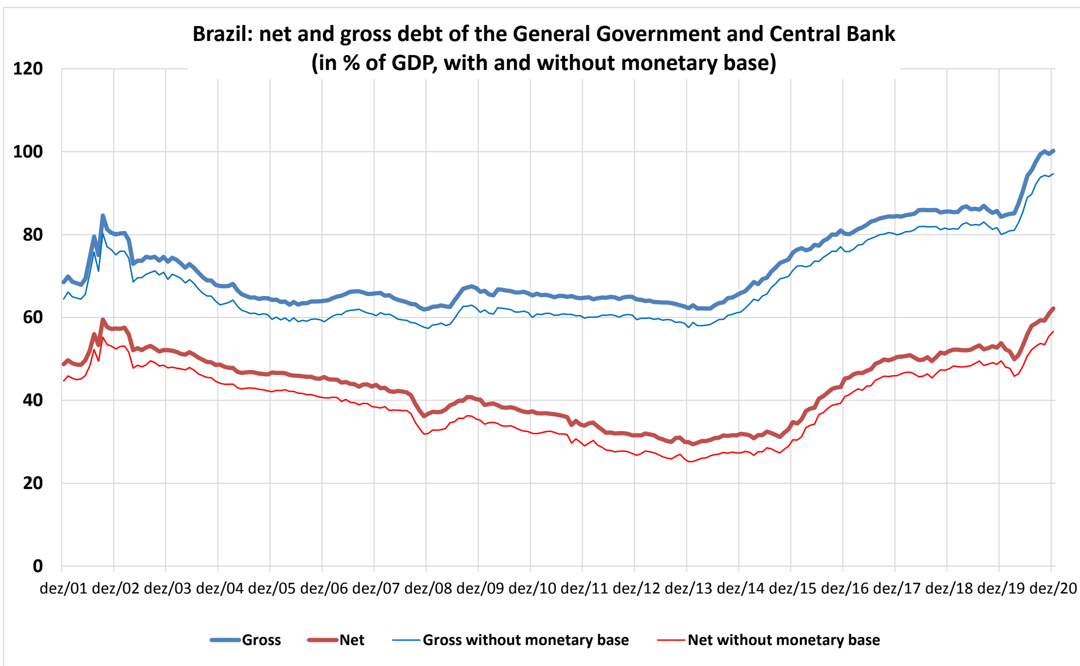
Based on (3.11) and assuming the real exchange rate is stable, the growth rate of foreign fixed-income assets necessary to keep the reserves-GDP ratio constant depends on the GDP growth rate and foreign inflation. Then, from the fiscal constraint, for the accumulation of foreign investments to have a zero impact on gross debt, we need:

$$\frac{1+i_{f,t}}{1+g_{Pf,t}} = 1 + r_{f,t} = 1 + g_{Y,t}. \tag{3.12}$$

In other words, for the balance-of-payments constraint (a stable reserve-GDP ratio) to have a zero impact on the fiscal restriction, the real interest rate earned on foreign reserves must be equal to the country's GDP growth. Since foreign reserves usually pay a low (frequently negative) real interest rate in foreign currency, it is more likely that $g_{Y,t}$ exceeds $r_{f,t}$, meaning that developing countries pay a fiscal cost to keep their international reserves stable in terms of GDP.

Moving again to the Brazilian case, figure 4 presents the General Government's and the Central Bank's net and gross debt. Including the monetary authority in the definition of government allows us to capture the international reserves' influence in public debt dynamics because, in Brazil, international reserves are in the Central Bank's balance sheet. Since part of the debt includes the monetary base, figure 4 also presents the series with and without money.

Figure 4: net and gross debt of the Brazilian General Government and Central Bank, in % of GDP



Source: Brazilian Central Bank and author's calculation.

The Brazilian government started to accumulate more international reserves in 2006, after paying down its debt with the IMF and a change of command in the Ministry of Finance.[4] The process continued until 2012, when the Brazilian Central Bank reached USD 350 billion in international reserves. Since then, the value has remained stable around USD 350 billion.

In terms of figure 4, the government accumulation of foreign exchange explains why Brazilian gross debt remained stable between 2005 and 2014, while net debt went down. The Brazilian government decided to buy "insurance" against international financial shocks, paying a high carry cost to do it (the domestic interest rate on government bonds was much higher than the interest rate paid on international reserves). Despite such a price, the choice

4 In March 2006, Guido Mantega replaced Antonio Palocci as Brazil's Minister of Finance.

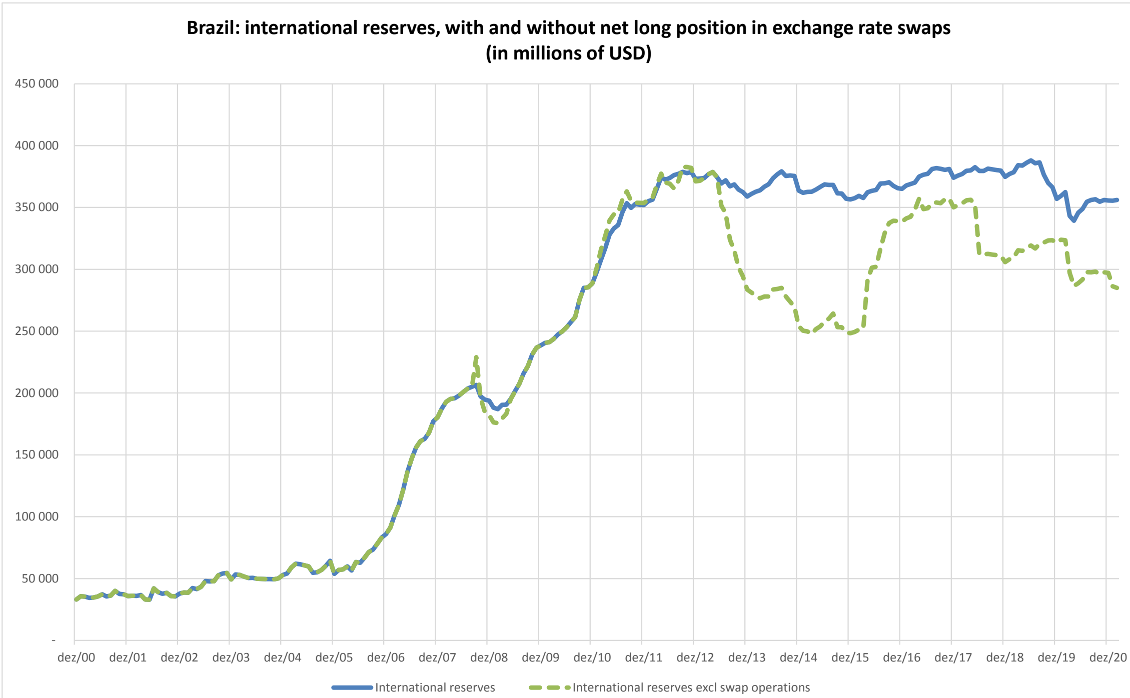
proved to be correct since it has allowed Brazil to wither substantial financial shocks – the Global Financial Crisis of 2008 and the Covid sudden stop of 2020 – without significant problems in its balance of payments.

The accumulation of foreign exchange allowed the Brazilian Central Bank to smooth the impact of global shocks through exchange-rate swaps in the local market. In this kind of trade, during periods of depreciation, the monetary authority pays the exchange-rate variation plus an interest rate and receives the Brazilian base interest rate (the SELIC rate). In periods of sharp appreciation, the Central Bank does the reverse operation. The exchange swaps are indexed to the exchange rate but settled in domestic currency, meaning the Brazilian Central Bank can never go insolvent because of it.

In practice, the Brazilian local exchange-rate swaps work as a domestic hedge against sharp fluctuations of the exchange rate and, since the late 2000s, the strategy has worked to reduce exchange-rate volatility and avoid speculative attacks to the Brazilian real, but sometimes with a high fiscal cost (Barbosa-Filho 2018). The Brazilian actions have also benefited from a swap agreement with the Federal Reserve, in which the US Central Bank acts as a lender-of-last-resort to some Central Banks, lending USD and borrowing the other part's currency at pre-determined interest rates and exchange rates.

But how important are exchange-rate swaps for Brazil? As shown in figure 5, in periods of financial distress, as in 2014-15, the Brazilian Central Bank went as "short" as USD 115 billion in local exchange-rate swaps, approximately one-third of the country's international reserves. Since the financial costs of the swaps count as net interest (paid or received) in fiscal accounting, they should be treated separately in any analysis of public-debt sustainability.

Figure 5: Brazilian international reserves with and without the Central Bank's net position in exchange-rate swaps, in millions of USD.



Source: Brazilian Central Bank

In terms of the model outlined above, the swaps' net cost should be added as a separate positive component on the right-hand side of (3.1). The size of such cost depends on the country's target for international reserves and how much of it can be used to smooth exchange-rate fluctuations, in addition to the local and foreign interest rates. The cost can be well described ex-post, but as anything related to exchange rates, it is hard to forecast ex-ante.

Conclusion

This note presented an accounting framework to describe public-debt dynamics when the government issue liabilities and buy assets in two currencies. The model used Brazil as a reference, but it can be used to describe the case of other developing countries that do not issue international reserve currencies. The fiscal accounting identities can be adapted to alternative theoretical closures or policy rules and the main results of the model are useful for a taxonomy of fiscal fragility or robustness. The annex presents the derivation of all equations shown above and a small-scale model for fiscal simulations.

References:

Barbosa-Filho, N.H. (2011): “International Liquidity and Growth”, SCEPA working paper, later published as a chapter in: Epstein, G. (ed), Financialization and the World Economy, London: Edward Elgar.

Barbosa-Filho, N.H. (2018), “Composição dos Juros Líquidos Pagos pelo Setor Público no Brasil: 2002-2017”. Brazilian Journal of Political Economy, vol.38, n.3, pp.548-558.

Barbosa-Filho, N.H. (2014): “Resultado primário, dívida líquida e dívida bruta: um modelo contábil”, em: Bonelli, R. e Veloso, F. (org), Ensaios IBRE de Economia Brasileira II, Rio de Janeiro: Elsevier.

Barbosa-Filho, N.H. (2021). Public debt dynamics with two currencies. Background Report to UNCTAD Trade and Development Report 2021. March.

Barro, R. (2020): “r minus g”, NBER Working Paper 28002.

Blanchard, O. (2004): “Fiscal Dominance and Inflation Targeting: Lessons from Brazil”. NBER Working Paper 10389.

Buiter, W. (2010): “Seigniorage”, NBER Working Paper 12919.

Escolano, J. (2010): A Practical Guide to Public Debt Dynamics, Fiscal Sustainability, and Cyclical Adjustment of Budgetary Aggregates. Washington DC: International Monetary Fund.

Godley, W. and M. Lavoie (2011): Monetary Economics: An Integrated Approach to Credit, Money, Income, Production and Wealth, London: Edward Elgar.

IMF (2014). Government Finance Statistics Manual, Washington DC: IMF, https://www.imf.org/external/Pubs/FT/GFS/Manual/2014/gfsfinal.pdf.

Walsh (2017): Monetary Theory and Policy, Cambridge MA: The MIT Press.

Public debt dynamics in emerging economies: an accounting model based on the Brazilian case

Recebido 04-out-22 **Aceito** 16-dez-22

Abstract *Public debt dynamics depends on the government's fiscal balance and the country's real interest rate and GDP growth rate. Money expansion also influence the process and, when the government can issue debt and but assets in foreign currency, the exchange rate is an important factor of public finance. Based on this view and inspired by the recent Brazilian case, this paper presents an accounting model of debt dynamics when the government's balance sheet has two currencies.*

Keywords *public debt, fiscal policy, emerging economies.*

Annex:

In the simplest case of equation (1.1):

$$D_t + \Delta M_t = -B_{p,t} + i_{t-1}D_{t-1}, \quad \text{(A1)}$$

$$\frac{D_t}{P_tY_t} + \frac{\Delta M_t}{M_{t-1}}\frac{M_{t-1}}{P_{t-1}Y_{t-1}}\frac{P_{t-1}Y_{t-1}}{P_tY_t} = -\frac{B_{p,t}}{P_tY_t} + i_{t-1}\frac{D_{t-1}}{P_tY_t}\frac{P_{t-1}Y_{t-1}}{P_tY_t},$$

and, therefore:

$$d_t = -b_{p,t} + \left(\frac{1+i_{t-1}}{1+g_{PY,t}}\right)d_{t-1} - \left(\frac{g_{M,t}}{1+g_{PY,t}}\right)m_{t-1} = -b_{p,t} + \left(\frac{1+r_t}{1+g_{Y,t}}\right)d_{t-1} - s_t \ .$$

In the general case of equation (3.1):

$$\Delta D_{h,t} + E_t\Delta D_{f,t} + \Delta M_t = -B_{p,t} + i_{Dh,t-1}D_{h,t-1} + E_t i_{Df,t-1}D_{f,t-1} - i_{Ah,t-1}A_{h,t-1} - E_t i_{Af,t-1}A_{f,t-1} + \Delta A_{h,t} + E_t\Delta A_{f,t}. \quad \text{(A2)}$$

Since

$$E_t = (1 + g_{E,t})E_{t-1}, \quad \text{(A3)}$$

$$A_{h,t} = (1 + g_{Ah,t})A_{h,t-1} \quad \text{(A4)}$$

$$A_{f,t} = (1 + g_{Af,t})A_{f,t-1} \quad \text{(A5)}$$

we can rewrite (A2) as:

$$(D_{h,t} + E_tD_{f,t}) - (D_{h,t-1} + E_{t-1}D_{f,t-1}) - (A_{h,t} + E_tA_{f,t}) + (A_{h,t-1} + E_{t-1}A_{f,t-a}) = -B_{p,t} - \Delta M_t + i_{Dh,t-1}D_{h,t-1} + i_{Df,t-1}E_{t-1}D_{f,t-1} + (g_{Ah,t} - i_{Ah,t-1})A_{h,t-1} + (g_{Af,t} - i_{Af,t-1})E_{t-1}A_{h,t-1} + g_{E,t}E_{t-1}(i_{Df,t-1}D_{f,t-1} - i_{Af,t-1}A_{f,t-1}) + g_{E,t}E_{t-1}(D_{f,t-1} - A_{f,t-1}) + g_{E,t}g_{Af,t-1}E_{t-1}A_{f,t-1} \quad \text{(A6)}$$

All equations in the main text come from normalizing (A6) by nominal GDP and simplifying notation.

To build an accounting model for simulation, it is simpler to start from (A6) and define the growth in total debt as:

$$F_t = \Delta D_{h,t} + E_t\Delta D_{f,t} \ . \quad \text{(A7)}$$

Next, define the amount of debt issued in domestic currency as a percentage θ_t of total debt issued:

$$\Delta D_{h,t} = \theta_t F_t. \tag{A8}$$

As a result, the evolution of the ratio of domestic and external debt are:

$$d_{h,t} = \frac{D_{h,t}}{P_t Y_t} = \frac{\Delta D_{h,t}}{P_t Y_t} + \left(\frac{1}{1+g_{PY,t}}\right) d_{h,t-1} = \theta f_t + \left(\frac{1}{1+g_{PY,t}}\right) d_{h,t-1}, \tag{A9}$$

and

$$d_{f,t} = \frac{E_t \Delta D_{f,t}}{P_t Y_t} + \left(\frac{1+g_{E,t}}{1+g_{PY,t}}\right) d_{f,t-1}. \tag{A10}$$

where f_t is (A7) normalized by nominal GDP. From (A6):

$$f_t = -b_{p,t} - \left(\frac{1}{1+g_{PY,t}}\right)\left[g_{M,t} m_{t-1} + i_{Dh,t-1} d_{h,t-1} + i_{Df,t-1} d_{f,t-1} + (g_{Ah,t} - i_{Ah,t-1}) a_{h,t-1} + (g_{Af,t} - i_{Af,t-1}) a_{f,t-1} + g_{E,t-1}(i_{Df,t-1} d_{f,t-1} - i_{Af,t-1} a_{f,t-1}) + g_{E,t} g_{Af,t} a_{f,t-1}\right]. \tag{A11}$$

The three remaining accounting equations describe the evolution of domestic and foreign assets, and the monetary base. In terms of GDP:

$$m_t = \left(\frac{1+g_{M,t}}{1+g_{PY,t}}\right) m_{t-1}, \tag{A12}$$

$$a_{h,t} = \left(\frac{1+g_{Ah,t}}{1+g_{PY,t}}\right) a_{h,t-} \tag{A13}$$

And

$$a_{f,t} = (1 + g_{E,t})\left(\frac{1+g_{Af,t}}{1+g_{PY,t}}\right) a_{ft-1}. \tag{A14}$$

Equations (A9) through (A14) gives us a system for six variables: $d_{h,t}$, $d_{f,t}$, $a_{h,t}$, $a_{f,t}$, m_t, and f_t. The exogenous variables can come from theoretical models, stylized facts, policy rules, or any other methodology. The list of exogenous variables is:

(i) the growth rate of GDP (g_Y),
(ii) the inflation rate, measured by the GDP deflator (g_P),
(iii) the growth rate of nominal money demand (g_M),
(iv) the change in the nominal exchange rate (g_E),
(v) the vector of interest rates paid and earned by the government (i_{Dh}, i_{Df}, i_{Ah}, and i_{Af}),
(vi) the pace of accumulation of domestic (g_{Ah}) and foreign (g_{Af}) fixed-income assets,
(vii) the share of debt issued in domestic currency (θ_t), and
(viii) the primary balance ($b_{p,t}$).

Because of the complexity of the eight factors outlined above, most fiscal analyzes concentrate on the primary balance, as if everything else remains constant. In practice, everything else does not remain constant. In both advanced and developing economies, fiscal adjustments often come from forces not directly linked to the government's primary budget.